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Recurrent nightmares?: measurement theory in de Sitter space

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Abstract: The idea that asymptotic de Sitter space can be described by a finite Hilbert Space implies that any quantum measurement has an irreducible innacuracy. We argue that this prevents any measurement from verifying the existence of the Poincaré recurrences that occur in the mathematical formulation of quantum de Sitter (dS) space. It also implies that the mathematical quantum theory of dS space is not unique. There will be many different hamiltonians, which give the same results, within the uncertainty in all possible measurements.

Keywords: AdS-CFT and dS-CFT Correspondence, Models of Quantum Gravity.

Contents

1. Introduction

A remarkable series of recent observations [1]–[7] (for a recent review see [8]), indicate that we live in an accelerating universe. It is quite tempting to speculate that the source of this acceleration is a small positive cosmological constant. If this is indeed the case, we would be living in an asymptotic de Sitter (AsdS) space. Some years ago, two of us argued [9, 10] that the quantum theory of such a universe has a finite dimensional Hilbert space.

Loosely speaking, a quantum version of the principle of general covariance would seem to suggest that a physical clock must be a quantum observable. This in turn suggests, that the clocks in an AsdS universe can only have discrete "ticks and tocks" and moreover it seems inevitable that these devices can only measure a finite total time interval. The spectrum of any observable in a finite dimensional quantum system is discrete and bounded.

At the present time, we only have a rigorous understanding of theories of quantum gravity with asymptotic timelike or null boundaries. In these systems, time evolution is governed by asymptotic symmetries, and we can imagine all clocks to be sitting at infinity. The measuring devices in such a universe can be considered to be arbitrarily large and classical, without having any effect on the scattering[1] experiments that they are measuring. This is not true in an AsdS universe. The purpose of the present paper is to address the question of what can actually be measured in such a universe.

In the absence of a complete proposal for the quantum theory of AsdS spaces, we rely on the intuition provided by the Wheeler-DeWitt formulation of quantum gravity to

[1] We use the word scattering as a shorthand both for the true scattering matrix in asymptotically flat spacetimes, and the boundary observables of asymptotically AdS spacetimes.

address these questions. We will find that the statement that physical clocks are operators in the physical Hilbert space of a general covariant system, is *not* a formal property of Wheeler-DeWitt quantization. Rather, we are led to a description of such a system in terms of a collection of different, more or less conventional, hamiltonian evolutions on the physical subspace. These evolution operators do not in general commute with each other. We have previously suggested that this lack of commutativity is the origin of Black Hole Complementarity [18, 19] and have generalized this notion to other spacetimes with horizons [11].

We will then argue that in a true quantum theory of cosmological spacetimes, continuous time evolution is probably an illusion. Furthermore, by studying actual measurements in AsdS spacetimes, we will argue that no reliable measurements can be made over arbitrarily long periods of time. We suggest that this means that the Poincaré recurrences of the mathematical formalism have no operational meaning [12, 13]. Finally, we argue that, as a consequence of the inevitable imprecision in measurements, there is no single mathematical theory of quantum AsdS spacetimes. Rather, there is a collection of different time evolution operators, which will make mathematical predictions for all possible measurements, with differences that are smaller than the fundamental imprecision of the measurements themselves. It is only in the limit of vanishing cosmological constant that precise mathematical statements have operational meaning. Equivalently, in this limit, there are *scaling observables* (in the sense of the renormalization group) that are universal, while many of the corrections to scaling are in principle unobservable.

The plan of this paper is as follows:

In the next section, we briefly review the arguments for a finite number of physical states in AsdS universes, and introduce a distinction between localizable states and horizon states. We then review the formal Wheeler-DeWitt quantization of gravity and its implication for the nature of clocks. In section 3 we review our understanding of quantum measurement theory. In particular we emphasize that it depends to a large extent on the locality properties of quantum field theory. In AsdS spaces, and on the horizons of black holes, there are many low energy states that cannot be simultaneously described by quantum field theory. While almost classical measuring devices *might* be constructed for other quantum systems with large numbers of states, we know too little about horizon states to describe a measurement theory for them, which does not involve a localized external observer. Thus, we will restrict our attention to localizable measurements. In section 4, we show that very simple estimates for localizable measurements, imply that the time for a quantum fluctuation to fuzz out the classical pointer positions of any localized measuring device in AsdS space is much smaller than the dS recurrence time. This is the central point of this paper. We also identify a set of measurements in dS space which asymptote to the S-matrix observables of asymptotically flat space, and explore the relation between these idealized observables and observations made by a localized timelike observer. Finally, we comment on the likelihood that time evolution in cosmological spacetimes is fundamentally discrete.

We should also mention that other authors have touched on the issue of the limitations of measurement in asymptotic de Sitter spaces [14, 15, 16].

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2. Infinite aspirations and finite reality

2.1 The phase space of AsdS gravity

Much of the recent literature on dS space is focused on the attempt to construct gauge invariant meta-observables, which are analogs of the boundary correlators of the AdS/CFT correspondence [28]. We believe that these attempts are misguided, both because these meta-observables are mathematically ill-defined, and because (and this is a central message of the present paper) physical measurements in AsdS spaces have (in principle) limited precision. In a sense we will try to make precise in section 4, this imprecision can be viewed as a lack of "gauge invariance" of real measurements.

One way to approach this problem[2] is to think of the phase space of classical gravity; the space of solutions with given boundary conditions. We are used to thinking about the space of solutions to a field theory by invoking the Cauchy-Kowalevska (CK) theorem to map it into the space of initial conditions on a fixed spacelike slice. We believe that in General relativity, this theorem is misleading because of the generic occurrence of singularities. We have very little knowledge about global existence theorems for GR. The CK philosophy leads us to to view GR as a field theory. Instead we believe (invoking a conservative version of the Cosmic Censorship conjecture) that generic solutions with scattering boundary conditions lead at most to spacelike singularities cloaked behind black hole horizons. The Bekenstein-Hawking formula for black hole entropy, and the parametrization of phase space by scattering data, then lead us to a *holographic* counting of the degrees of freedom of GR.

Let us apply these ideas to AsdS spaces. The authors of [28] have suggested, in analogy with the AdS/CFT correspondence, that one define observables in the semiclassical approximation by looking at the response to probes on $\mathcal{I}_{\pm}$. Intuitively, one is inserting "particles" on the past and/or future boundaries of dS space. A particle is an infinitesimal classical disturbance, but actually carries a discrete minimal energy (for fixed wave number). The calculations suggested in [28] correspond to a perturbative solution of the classical equations with fixed sources on the boundary. One can investigate the effect of minimal particle energies by considering the phase space of the full nonlinear equations.

Consider generic boundary conditions on either of $\mathcal{I}_{\pm}$. The present authors (and probably a number of relativists before them) have conjectured that apart from a compact submanifold in boundary condition space, such solutions have a spacelike singularity of Big Bang or Big Crunch type.[3] Intuitively,[4] if we inject too much energy at the boundary, the energy density reaches that at which black holes as large as the spatial sphere form, before the sphere shrinks below some finite size. We will call this minimal size the "radius at the singularity". To make a more precise definition, we could define the radius at the

[2]Which is different from the arguments we have previously given in published work.

[3]G. Horowitz has informed us that he and N. Itzhaki have found a proof of this result in certain circumstances.

[4]We work in global coordinates. Energy refers to the nonconserved global time translation operator. Energy density is covariantly conserved in the dS background geometry. We can use it at a good physical parameter to the extent that the dS background is undisturbed by the perturbation we are studying.

singularity to be the point at which geometrical quantities suffer deviations of order one from their dS values. The approximate dS geometry then defines what we mean by this radius. Note that unlike the case of asymptotically infinite universes, we cannot rely on the localization of the perturbation to guarantee that the asymptotic geometry is unaffected. The spatial sections of dS space are compact.

The singularities for different solutions, will occur at different values of the radius at the singularity, because black holes of different sizes will be formed. Thus, we believe that, apart from the compact subspace of the naive asymptotic phase space, for which global solutions are nonsingular or contain black holes smaller than the minimal dS radius, the existence of solutions depends on the resolution of singularities. It seems clear to us that solutions in which the singularity occurs should not be considered part of the same theory as those which are nonsingular. It is unclear whether the singular solutions are related to a well defined quantum theory, and if so, whether singularities which deviate from dS space at different radii can be viewed as part of the same system.

The consequence of this point of view is that the space of solutions of GR, which asymptotically dS in both the past past and future, is compact, and upon quantization , should lead to a finite number of physical quantum states. We think of this as the quantum theory of idealized dS space. Among the future AsdS spaces with a Big Bang singularity, we imagine that there is at least one subspace, which defines a sensible quantum theory that describes the Universe we observe. In the rest of this paper we will explore the idealized dS quantum theory, since the issues we address have nothing to do with the Big Bang.

2.2 Wheeler DeWitt quantization

The residual gauge invariances of the synchronous gauge in General Relativity are the freedom of choice of the initial spatial slice, and spatial reparametrizations of that slice. Correspondingly, there are two classes of constraints:

$$\mathcal{H}_i(\mathbf{x}) = 0 \tag{2.1}$$

$$\mathcal{H}(\mathbf{x}) = 0\,. \tag{2.2}$$

The first set merely state that physical quantities are diffeomorphism invariant on the spatial slice. They are kinematical, and easy to solve. The second constraint is the Wheeler-DeWitt equation, which is dynamical.

The essential point we want to make is simplified, if we further choose the gauge, so that only a global reparametrization of time is allowed. Then the Wheeler-DeWitt constraint becomes, upon formal canonical quantization, a single, hyperbolic differential equation:

$$[G^{ij}(\phi)\partial_i\partial_j + U(\phi)]\Psi = 0\,. \tag{2.3}$$

The ϕ are the canonical coordinates of the system and the DeWitt metric G_{ij} has signature $(1, N)$. The wave function Ψ might be a finite dimensional vector of wave functions, and the potential U might be a matrix in this finite dimensional space. The simplest example of a time reparametrization invariant system is the relativistic particle. In this case the Wheeler-DeWitt equation is just the Klein-Gordon equation. String theory teaches us that in order for this equation to make sense at the quantum level, one must generally include contributions from Fadeev-Popov ghosts in the Wheeler-DeWitt operator. However, it is not our intention to propose Wheeler-DeWitt quantization as a rigorous approach to a realistic theory of quantum gravity, so we will ignore this technicality.

The space of wave functions on which the Wheeler-DeWitt operator acts is not a Hilbert space. It is only the solution space of the equation which is to be interpreted as the physical Hilbert space of the system. Solutions of the WD equation are parametrized, according to the CK theorem by the values of Ψ and its normal derivative, *on a spacelike surface in the space of variables* ϕ^i. In a correct quantum theory, this solution space, or some subspace of it, will carry a positive definite inner product. Note that the solution of the WD equation will define a one parameter set of vectors in this physical Hilbert space. The inner product should be defined in such a way that it does not depend on which spacelike slice in field space it is evaluated on. Then the one parameter set of vectors will be related by unitary evolution in the Hilbert space.

One often tries to give an intuitive description of this mathematical procedure by arguing that the physical meaning of time reparametrization is that clocks must be chosen to be physical objects. The choice of gauge, which reduces the full-blown WD equation to a hyperbolic PDE with only one negative eigenvalue of G_{ij} is "a choice of physical clock". Note however that the clock variable is not a quantum variable in the physical Hilbert space. Rather, it defines a one parameter family of states in the physical Hilbert space, much like a classical time variable.

In fact, the problem of constructing a positive metric subspace of WD solution space can only be solved in a general manner in the semiclassical approximation [24]. Here one chooses a class of solutions of the WD equation of the form (the justification for this form depends on the details of the WD equation)

$$\Psi \sim e^{i\frac{S(t)}{\epsilon}}\psi(t, \phi^k) \,, \tag{2.4}$$

where ϵ is a parameter whose small size justifies the WKB approximation. t is a particular combination of the coordinates ϕ^i, which parametrizes a spacelike slicing of a particular classical solution of the system ψ satisfies a time dependent Schrodinger equation

$$i\partial_t\psi = H(t)\psi \tag{2.5}$$

and the solutions of this equation have a well known time independent, positive, scalar product, if $H(t)$ is hermitean.

In general, for a given classical solution of the system, there may be many ways to choose the time slicing. Each of these defines a different unitary evolution on the same Hilbert space. There is no reason for these different evolution operators to commute with each other, at any time. This leads to a quantum complementarity between the observations of observers using different classical coordinate systems. In a quantized theory with general coordinate invariance there is no reason to expect to be able to discuss simultaneously, the observations of two observers using different coordinate systems. Two of the authors have argued that this is the origin of Black Hole Complementarity [11].

Beyond the semiclassical approximation, there are apparent problems with the straightfoward WD approach. The Klein-Gordon example provides some insight into these problems. In that context they have to do with the mixing of positive and negative frequency modes (in general the scalar product is positive only on one of these subspaces) and the necessity for second quantization. There is no satisfactory resolution of these problems for quantum gravity in more than two dimensions. For this and other reasons (principally the spatially local parametrization of phase space — which seems to us incompatible with holography), we do not believe that the WD equation provides a correct definition of a quantized theory of gravity.

However, it seems extremely plausible that certain features of WD quantization will survive in a well defined quantum theory of gravity. The existence of many equivalent but complementary time evolution operators (which generally do not have a time independent hamiltonian) would seem to be a general consequence of combining quantum mechanics with general covariance. We will argue later that there may be contexts in which time evolution is discrete rather than continuous, but that does not change the general structure in a drastic way.

3. Measurement theory and locality

At the heart of the quantum theory of measurement is Von Neumann's theorem that unitary evolution can take a product state

$$|A_0\rangle \otimes \sum c_s|s\rangle$$

into an entangled state

$$\sum c_s|A_s\rangle \otimes |s\rangle \, .$$

We think of $|s\rangle$ as a complete orthonormal basis of the quantum system to be measured, while the states $|A_i\rangle$ are states of the apparatus. Von Neumann shows that the apparatus states in the entangled state can be chosen orthonormal. Indeed, unitary evolution can take any state in the tensor product Hilbert space into any other with the same norm. If the $|A_s\rangle$ are orthonormal, expectation values of an arbitrary system operator O_s in the entangled state, are given by

$$\langle O_s\rangle = \sum |c_s|^2 \langle s|O_s|s\rangle \, . \tag{3.1}$$

This is the expression for the classical expectation value of a quantity in a probability distribution $P_s = |c_s|^2$. Quantum interference has disappeared. We say that we have measured those operators of the system, which are diagonal in the $|s\rangle$ basis.

The problem with this argument is that the apparatus itself is a quantum system and we can imagine reexpressing this same entangled state in terms of another orthonormal basis for apparatus states. Then the same measurement can be interpreted as measuring something else. In the case where the c_s are all equal we can interpret the same time evolution as a measurement of operators which do not commute with the $|s\rangle$ projectors. Very clear discussions of this problem can be found, for example, in [21, 22].

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Much recent literature on this problem has focused on the resolution of this ambiguity, "the choice of pointer basis", by environmentally induced superselection, i.e. the interaction of the apparatus with an unobserved environment. It is often stated that the size of the apparatus cannot by itself resolve this ambiguity.

Our own understanding of the measurement problem is somewhat different , and does depend on the size of the Hilbert space of the apparatus. At present it also depends on a good approximate description of the physics of the apparatus by (perhaps cut-off) local field theory. As orientation for this discussion let us remind the reader of the discussion of spontaneous symmetry breaking in field theory (or more general occurrences of moduli spaces of vacua). In particular we want to draw attention to the well known fact that one is *not* allowed to make superpositions of states which have different constant expectation values of a scalar field. The essential reason for this is locality. The Hilbert space of a field theory is constructed by acting with local operators on a single Poincaré invariant vacuum state. States with different Poincaré invariant expectation values cannot be related to each other by local operations.

This principle is valid for cutoff field theories, and it even has implications for large but finite systems. In the latter, states with different expectation values can communicate with each other via local operations, but their overlaps are of order e^{-V} where V is the volume of the system in cutoff units. Furthermore, although in systems with a symmetry, states with expectation values of symmetry breaking fields are not energy eigenstates, the quantum tunneling amplitudes that convert such an initial state into a properly symmetric eigenstate are exponentially small. Thus, in a large but finite system there are long lived states that have expectation values of quantities that are macroscopically distinguishable. In fact such states are members of huge subspaces of the Hilbert space which have almost the same value for these macroscopic operators. The expectation values are thus robust in the sense that many local operations on the system do not change them (or change them by inverse powers of V). Of course, local operations that dump huge amounts of energy onto a finite system *can* change macroscopic expectation values,[5] but there is a wide range of perturbations which do not.

Our understanding of measurement theory is based on such macroscopic observables in local field theory. For us, a measuring device must be huge in microscopic terms, and have many more states than the system it is measuring. It must have macro-observables like the expectation values of scalars in field theory, which take on almost the same value in a finite fraction of the states of the apparatus. The overlap between states with different values of the macro-observables must be very small, and tunable to zero as the size of the apparatus goes to infinity. Large classes of perturbations of the apparatus must leave the subspaces with fixed macroobservables invariant, with very high probability. Cutoff local field theory for a finite volume system provides us with many explicit examples of systems with these properties. There may be other examples, but to our knowledge there does not exist a systematic theory of nonlocal measuring devices.

[5]For example, inserting a tiny, but macroscopic, amount of antimatter into any piece of laboratory equipment, will disrupt its ability to measure anything.

Notice several points about our description: the exact orthogonality of the $A_s\rangle$ assumed in Von Neumann's discussion is unnecessary. It is sufficient that the overlaps of different pointer states are exponentially small at larger pointer size. We do not then recover an idealized measurement as expressed in the axioms of the Copenhagen interpretation. The expectation values in the entangled state do not reproduce the classical probability sum rules exactly but only up to exponentially small corrections. The volume that appears in these exponentials is that of the pointer rather than of the whole apparatus. Finally, these remarks imply that for a finite apparatus the whole concept of measurement is only an approximation. Not only are the classical probabilities achieved only in an approximate manner, but the measurements are not exactly robust. Given a long enough time, quantum fluctuations of the pointer will nullify the utility of the apparatus as a measuring device. Ideal Copenhagen measurements are only achieved in the limit of infinite apparatus.

The above discussion ignores the interaction of the apparatus with its environment — that is the effects which are taken into account for environmentally induced superselection. These are important for ordinary experiments, but not, we believe, for the idealized "maximally accurate" experiments in dS space that we will study in the next section. That is, while we accept the fact that environmentally induced superselection may be quantitatively important in understanding the behavior of many everyday measuring devices, we do not believe that environmental decoherence is necessary to the construction of a sensible measurement theory. In particular, in the next section we will discuss idealized measurements in dS space in situations where the measuring apparatus does not seem to have much of an environment. We claim that large, approximately local, devices are capable of making almost ideal quantum measurements without the benefit of an environment. A measurement consists of entangling a complete set of basis states of the measured system with macroscopic, approximately orthogonal, states of the apparatus. In the limit of infinite apparatus size, macroscopic states become orthogonal superselection sectors. It is possible that other large systems can also act as measuring devices. In particular, black hole and cosmological horizons are repositories of large numbers of quantum states. It is possible, but beyond our current capabilities to prove, that horizons or pieces of them, can act as measuring devices.

4. Measuring de Sitter

In any accurate measurement, one wants to minimize the effect of the apparatus on the measured system. We want to correlate the states of the system prepared in some experiment, with macroscopic apparatus states, without having the apparatus interfere with the experiment. In a theory of gravity this is difficult because any apparatus has a long range, unscreenable, interaction with any experiment. The newtonian estimate of the strength of this interaction is $G_N M/R$, where M is the mass of the apparatus, and R is its minimal distance from the experiment. We could try to minimize this interaction by making M very small, but this interferes with our ability to make the large, almost classical, devices required by measurement theory. The only alternative, is to make the distance R very large.

This simple estimate is a powerful hint that the only well defined mathematical observables in a quantum theory of gravity should be scattering data. That this is indeed the case in string theory is well known, and has led to a lot of angst about the appropriate observables in dS space. We should emphasize precisely what we mean by scattering data, since , by abuse of language one often considers ordinary laboratory experiments to be measuring the S-matrix. This is clearly not true. One doesn't wait an infinite amount of time or put one's detector at infinite distance in a real experiment. In quantum field theory, real experiments are more appropriately modelled in terms of the response of the system to localized sources. The S-matrix is a limit of such amplitudes when the sources are taken to infinity, as expressed in the Lehmann-Symanzik-Zimmerman formula. Standard estimates show that for realistic experiments, the difference between the localized amplitudes and the real S-matrix is negligible.

Although the gravitational effects of the detector on any reasonable experiment, are for the time being way beyond our capabilities to measure, they are there in principle. Furthermore, the precise gravitational effect depends on the nature of the detector. Thus, in a quantum theory of gravity we do not know how to construct the analog of localized measurements that correspond to precisely defined mathematical quantities. The only way to eliminate the gravitational effect of the detector is to remove it to infinity, and we can only do this if spacetime is infinite.

Our approach to this problem [11] will be to rely on the idea that asymptotically flat space is a limit of dS space. Thus, there should be mathematical quantities in any quantum theory of dS space, which approach the flat space S-matrix as the cosmological constant goes to zero. As in any limiting situation, there may be ambiguities in what these quantities are for finite values of the cosmological constant. Our strategy in this paper will be to find a physical definition of these approximate S-matrix quantities. We will find that they do not correspond to observations that can be made by local observers in dS space. We will then try to elucidate the connection between these two kinds of measurement. Throughout our discussion, we will ignore the special complications of four dimensions, where the flat space S-matrix does not exist because of the infrared problems of graviton bremstrahlung.

Measurement theory in dS space is caught in a web of contradictory requirements. To minimize quantum fluctuations in our apparatus, we must make it large. This requires us to move it far away from the experiment, but the existence of a cosmological horizon makes it difficult to do this. A related problem is the putative existence of only a finite number of states in dS space. Any measurement in such a finite system consists of a split of the states into those of a "system" and those of an apparatus. A bound on the total number of states means that the apparatus cannot approach the classical behavior required of an ideal measuring apparatus.

There is a further problem, which we encounter if we attempt to make our apparatus stationary with respect to the frame of the experiment. It then becomes impossible to move it far away. If we consider a sequence of stationary measuring devices, closer and closer to the horizon of the experiment, the later devices in the sequence find themselves in a hotter and hotter environment. Eventually they are destroyed by Hawking radiation.

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This indicates that the best experiments one can do in dS space are those performed by machines freely falling with respect to the experiment, machines that will eventually go outside the experiment's cosmological horizon. We consider experiments whose "active radius" remains finite as the dS radius goes to infinity. By active radius, we mean the radius outside of which the experiment can (with some accuracy) be said to consist of freely moving particles in Minkowski spacetime. Of course, if we really want to get precise answers, we have to let the active radius go to infinity with the dS radius.

We consider machine trajectories whose closest approach to the experiment goes to infinity as a power, R_{dS}^{α}; $\alpha < 1$, of the dS radius. Such machines can be taken to have a mass μ, which scales like $\mu \sim R_{dS}^{\beta}$. As long as $\beta - \alpha < 0$, the gravitational effect of the detector on the experiment, will vanish as the dS radius goes to infinity. Thus, in this limit we can make very massive, freely falling machines, whose gravitational effect on the experiment is negligible.

Further constraints come from the requirement that our machines are approximately described by local field theory. We emphasize that this requirement would not be fundamental if we could learn how to manipulate the internal states of black holes or the cosmological horizon to make measuring devices. If we rely on "current technology" then the best we can do is to use the high energy density of states of a field theory to make our measuring devices. These then have an energy and entropy density,

$$\rho \sim T^4 ; \qquad \sigma \sim T^3 . \tag{4.1}$$

The requirement that a machine made out of such conformal stuff not collapse into a black hole, restricts its total entropy to scale like $R^{3/2}$ where R is the linear size of the (roughly isotropic) machine. This is the same power law that one obtains for the entropy of a starlike solution of the Tolman-Oppenheimer-Volkov (TOV) equations [25, 26, 27] We have seen above that R can scale at most like a power of R_{dS} less than α.

Now the crucial point is that the tunneling amplitudes between macroscopic state of the apparatus are of order N_{ap}^{-p} where p is an exponent of order one and N_{ap} the number of states of the apparatus. This is a consequence of the fact that if we write the Hilbert space for a local system as a tensor product of spaces of localized states, then by definition two macrostates different from each other by an amount of order one in each of a finite fraction of the factors. The total number of states is

$$N_{ap} = n^k \tag{4.2}$$

where n is the number of local states and k the number of localized factors. The overlap between two macrostates is

$$\langle \Psi_1 | \Psi_2 \rangle \sim \prod_{i=1 \ldots ak} o_i , \tag{4.3}$$

where a is a fraction of order one and each o_i is a local overlap, a number less than one, but of order one. This is of order N_{ap}^{-p}, as claimed.

Putting together these results, we find that the tunneling time between different macrostates representing different pointer positions on our measuring device, is of order $e^{cR^{3/2}}$, where R is the size of the device. It is clear that R can scale at most like some power of the dS radius. From the scaling law of the TOV equations, the size scales like the mass: $R \sim R_{dS}^{\beta}$, where as shown above, $\beta < \alpha < 1$ if we want to avoid interference between the device and the experiment. *Thus, the tunneling time between macrostates of any sensible local measuring apparatus in dS space, is much shorter than the Poincaré recurrence time.* Indeed, for a dS radius corresponding to the observed acceleration of the universe, the recurrence time is so long that it is essentially the same number expressed in seconds as it is in units of the tunneling time of the largest possible apparatus in dS space. As a consequence, no local measurement (which, in our current state of ignorance of horizon physics means no measurement we know how to describe) can be made over time periods remotely comparable to the recurrence time. A mathematical description of dS space may contain the phenomenon of Poincaré recurrence, but there is no apparent way to test this mathematical conclusion by experiment. Quantum fluctuations in the measuring device will wipe out all memory of measurements long before the recurrence occurs.

Environmentally induced decoherence could lengthen the time period of utility of a quantum measuring device. This is certainly an important quantitative effect for normal laboratory equipment. Indeed, it has been claimed that the decoherence due to interaction with the solar wind at the orbit of Saturn is necessary to explain the classical rotational behavior of one of its satellites, Hyperion [23]. However, the experiments we are contemplating take place in an environment much more rarefied than interplanetary space. We are discussing observables in dS space that approach the S-matrix of an asymptotically flat spacetime in the limit of vanishing cosmological constant. The S-matrix describes the behavior of finite numbers of particles in an infinite empty universe. There is no environment around to help the idealized "S-matrix meters" decohere, and we must rely on the superselection inherent in the behavior of these large local machines to perform measurements.

Finally we note that even if we could build measuring devices from the cosmological horizon states, we would find that the tunneling time between pointer macrostates of the horizon would be much smaller than the recurrence time, since such states could at most carry a finite fraction of the total entropy.

It is obvious that, given the size of the cosmological constant implied by observations, these restrictions on measurement have no practical content. The technological restrictions implied by our own local nature will, forever, make the measurements we can actually do much less accurate than these bounds would allow them to be. The significance of these bounds is to call into question the utility of discussing the recurrence time scale and to show that no mathematical quantum theory of AsdS space can be unique.

Indeed, it is clear that there must be many hamiltonian descriptions of the same system, whose predictions are sufficiently similar that no conceivable experiment could distinguish them. The complete set of observables in a quantum system with a finite number of states, can only be measured by an infinite external measuring device. Any self measurement in such a system requires a split of the states into observer and observed, and an intrinsic uncertainty in measurements as a consequence of the quantum fluctuations of the measuring device. If we cannot measure all the mathematical consequences of a given definition of observables, then there will be many different mathematical theories which have the same consequences for all measurements.

We would like to conjecture that the existence of many indistinguishable hamiltonians for AsdS space is a kind of gauge equivalence, related to the infamous Problem of Time in the WD quantization of gravity. The Problem of Time in a closed universe is the absence of a preferred definition of time evolution. Different choices of time evolution are formally related by gauge invariance. But, in a closed universe, there are no gauge invariant operators. Thus, as we discussed in the section on WD quantization, it would appear that a theory of quantum gravity in a space that does not have infinite extent must be discussed in terms of a collection of different time evolution operators. In principle these are supposed to be "gauge equivalent". In classical GR this gauge equivalence is straightforward, but in the quantum theory there is every reason to suppose that the evolution operators corresponding to different observers do not commute with each other. Thus, in principle the same physical system can be viewed through different semiclassical lenses that are not quantum mechanically compatible with each other. In [11] we suggested that this was a way of understanding Black Hole Complementarity and generalized the notion to any spacetime with a horizon.

Our discussion of measurement theory in a finite system, leads to a similar conclusion: different choices of hamiltonian, but now with indistinguishable consequences for semiclassical measurements. The Problem of Time is more general and involves gauge equivalence between observers whose experience *is* measurably different. For example, the principle of Cosmological Complementarity [11], says that in the late stages of our AsdS universe, the observations of people in the Sombrero galaxy are gauge equivalent to our own. Each of us views his own physics as local, and his erstwhile companion's physics as a quantum fluctuation on his event horizon. But there are also changes of coordinate system which do not change semiclassical observations in an measurable way. We conjecture that the amiguity in the definition of the static dS hamiltonian may be related to such a coordinate ambiguity.

4.1 Local observations

The measurements we have been discussing so far are not realistic measurements from the point of view of a local observer in dS space. The machines that measure initial and final states of particles at different angles in our scattering experiment are outside each other's event horizons. There is, until we take the $\Lambda \to 0$ limit, no single observer who can collect all of this information. We now want to discuss actual measurements by an observer gravitationally (or otherwise) bound to the experimental apparatus (the accelerator, not the detectors). As far as we have been able to tell, the most accurate and nondestructive measurements this observer can make on his experiment consists of receiving information from the free falling "S-matrix meters" that we have discussed previously. That is, one must equip those devices with a signaling mechanism, which can send photons or other messenger particles back to the observer at the center of the experiment's horizon volume.

It is clear that the local observer can garner *less* information than is available to the freely falling machines. There are two separate effects here. First of all, as the machines approach the horizon, and their disturbance of the experiment goes to zero, the photons they emit are redshifted. For any fixed lower frequency cutoff on the local observer's photon detection system, there will come a time when signals from the freely falling machine are undetectable. Furthermore, the photons sent to the center from the freely falling machine are *accelerated*. Thus, they will experience interaction with the Hawking radiation. As the machine approaches the horizon, this radiation gets hotter and denser. These random interactions will tend to destroy the information contained in the signal sent to the local observer. Thus, we can understand, in terms of relation between the S-matrix meters and local observations, the fact that the local observer finds the universe to be in a mixed state, while the S-matrix observations retain their quantum mechanical purity.

A final constraint on the accuracy of measurements made by a local observer comes from the fact that he is immersed in thermal bath at the dS temperature. Although this temperature scales like $1/R_{dS}$, the probability for a catastrophic burst of photons or the nucleation of a black hole which would engulf him is nonzero. The timescale between such events is, again, much less than the recurrence time. This phenomenon also occurs for the freely falling S-matrix meters. However, since their size is taken to scale to infinity with the dS radius, the effect can be made small in the limit of infinite radius. The predictions for the limiting Minkowski theory will be precise quantum measurements.

4.2 Discrete time?

This subsection is not strongly correlated with the rest of the article. We include it because our original speculations about physical clocks suggested a discrete and finite time evolution. We have already seen that, although the mathematical formalism appears to be compatible with arbitrarily long times, we cannot set up measuring devices within the system that could measure correlations over (much less than) a Poincaré recurrence time.

It is certainly true that the static hamiltonian in dS space has a maximal eigenvalue, corresponding to the Nariai black hole. Standard discussions of the energy time uncertainty relation then imply that we cannot set up time measurements of intervals shorter than the inverse of this energy. In a certain sense then, our intuition is correct.

A more formal argument that time evolution is discrete comes from our proposal for describing quantum mechanical Big Bang cosmologies [11, 17]. Here, the results of all possible experiments done by a local observer are described by a sequence of Hilbert spaces of (multiplicatively) increasing, finite, dimension. A given Hilbert space describes all experiments that could have been done in the causal past of a given point in spacetime. Time evolution along a particular timelike trajectory must be compatible with the state of the system breaking up into tensor factors at earlier times, with one factor describing all data that could have been accumulated inside of the particle horizon at a given point on the trajectory. Since Hilbert space dimensions are discrete, there is a natural discrete breakup of the time interval.

If we look at typical FRW cosmologies, the Fischler Susskind Bousso (FSB) area of the particle horizon scales like a power of the cosmological time. Thus, the minimal increase in area, corresponds to a smaller and smaller increase in cosmological time, as time goes by. The time resolution becomes finer and finer, and goes to zero if the particle horizon eventually becomes infinite.[6] However, in AsdS space, there is an ultimate limit to this increase in resolution, because the particle horizon ceases its growth.

[6]Note that this is in perfect accord with arguments based on the energy-time uncertainty relation. An

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5. Conclusions

We have argued that a universe which asymptotically approaches dS space presents a serious challenge as to what is observable within that universe and how precise the measurement of such an observable can ever be.

We have shown that the essential limitation arises from the existence of an event horizon and the ability to describe AsdS space by a finite dimensional Hilbert space. In a nutshell, the necessity to separate the measuring device from the experiment such as to minimize the gravitational interaction, as well as availability of only a finite number of states to build detection devices, limits in principle the time period during which a measurement is "reasonably" sharp. The time after which the pointers on any conceivable local measuring device suffer uncontrollable quantum fluctuations is much shorter than the Poincaré recurrence time.

One of the consequences of these considerations not discussed in the text is that an inflationary epoch, for example due to a slowly rolling inflaton, in an AsdS universe is limited to be much shorter than a recurrence time. The reasons for such limitation is again the availability of a finite number of states to describe the dynamics of a scalar field. The quantum fluctuations of the inflaton field will wipe out the classical slow roll on a time scale shorter than the recurrence time. Therefore, although one can write down classical cosmologies in which an arbitrary number of e-foldings of inflation are followed by an AsdS era with any value of the cosmological constant smaller than the inflationary vacuum energy, many of these models will be ruined by quantum fluctuations.

It may be possible to model such dynamics by imposing a q-deformed algebra on the creation and annihilation operators of the inflaton, where the deformation parameter is the N-th root of unity and N is an integer commensurate with the finite number of the states. It seems clear that, for the value of the cosmological constant consistent with observations, the constraints on inflationary models are very mild. Even 10^{10} e-foldings of conventional slow roll inflation would be compatible with the constraints of *our* AsdS universe.

Our considerations suggest that any mathematical description of AsdS space is of limited utility. That is, there will be many different mathematical theories compatible with all possible experimental measurements, within the fundamental limitations on experimental accuracy. Furthermore, although a mathematical theory of AsdS space may describe arbitrarily long time scales, the recurrence time puts an upper bound on the operational meaning of any such description. Within the limits of "current technology", in which measurement theory is based on local devices, the actual operational limits on measurement time scales are dramatically smaller than the recurrence time. Only if one could learn to exploit and manipulate properties of horizon microstates could one approach measurements that would be robust over a recurrence time.

Separate arguments suggest a fundamental limit on the resolution of time measurements in an AsdS space.

observer with a larger particle horizon can access states of higher energy and therefore has a finer time resolution.

It is possible to take a pessimistic view of these limitations. In some sense they mean that if spacetime is AsdS, then the ideal goal of the theoretical physicist viz., to obtain a mathematically precise Theory of Everything is unattainable. One might be tempted to view this as a philosophical argument against the interpretation of current observations in terms of a cosmological constant. We reject such pessimism and its associated philosophical prejudices. In fact, the idealistic goal of a mathematical Theory of Everything seems a bit naive to us. Physics was developed as a way of explaining the behavior of isolated systems. The idea of experiments and ideally precise measurements presumes a precise, or at least infinitely refinable, separation between observer and observed. Quantum mechanics, which must (in our view) be interpreted in terms of infinitely large measuring devices, makes this idealization even more difficult to achieve. It seems almost inevitable that these mathematical idealizations should fail when applied to the whole universe, at least if it has a finite number of states.

The question of whether the universe has a finite or infinite number of states should be answered by experiment rather than philosophical prejudice.[7] Classically, one can build models of scalar fields with metastable minima, which can mimic an arbitrary cosmological constant for long periods of time, but then relax to an infinite FRW future. In such models, it makes sense to talk about times much longer than the Poincaré recurrence time of the temporarily dS universe. Indeed, this is quite analogous to our interpretation of observations of the Cosmic Microwave Background in terms of super-horizon size fluctuations of an inflationary universe. If what we are experiencing today is a second period of temporary inflation, it is not clear that measurements made by local observers prior to the end of inflation will be able to distinguish such a model from a true cosmological constant. Such local measurements would be subject to the constraints discussed in this paper. It would appear that only observers living after the Second Inflationary Era ends could verify the existence of Poincaré recurrences. Krauss and Turner have discussed the difficulty of determining the fate of the universe by cosmological observations in [29].

However, if the conjecture of [10], relating supersymmetry breaking to the cosmological constant, and the existence of a true cosmological horizon with a fixed finite number of states is correct, then we will be able to interpret the measurement of superpartner masses as a verification of the fact that the universe is finite. Such a verification would imply that a mathematical Theory of Everything did not exist and would signal that we have reached the limits of theoretical physics. Given the apparent value of the cosmological constant, these limitations are of no practical relevance, nor will they ever be. We have stressed that the imprecision implied by a finite number of states goes away in the limit that the cosmological constant goes to zero. In fact, the problems that we have been discussing go away exponentially with the cosmological constant and would not seem to pose a problem for e.g. computing elementary particle masses to any accuracy we might desire to know them. Philosophical reservations about such a view of the world seem to us to have little relevance to the real enterprise of physics. This has always been to give a

[7] The annoying question of what fixes the number of states if it is finite, cannot be answered in an experimentally testable way. Perhaps a sufficiently elegant theoretical answer to this question would calm our uneasiness about this point.

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simple explanation of the regularities *we* see in nature. Even with an extremely optimistic estimate of what our observational capabilities might be in the distant future, there is no danger that the fundamental limits to observation in AsdS spacetimes will ever effect the outcome of a real measurement.

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